Solving liveness problem for marked nets by exhaustive coverability trees*

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Abstract It is well known that a coverability tree of a Petri net cannot solve reachability and liveness problems of the net because using symbol ω (infinity) may lose some information. A solution to this problem is presented for a special kind of Petri net, marked net. With the combination of ω and the increasing/decreasing information of token number, a new kind of coverability tree of marked nets, called exhaustive coverability tree (ECT), is proposed. It is shown with an example that an ECT can be used to detect deadlock.

Keywords: **coverability trees, deadlock, marked nets.**

The coverability tree (CT) method is one of the useful analysis methods for Petri net. However, reachability and liveness problems cannot be solved by a CT alone. Consider two place/transition nets (P/T-nets) N_1(Fig. 1(a)) and N_2(Fig. 1(b)). Though their structures and reachability sets are

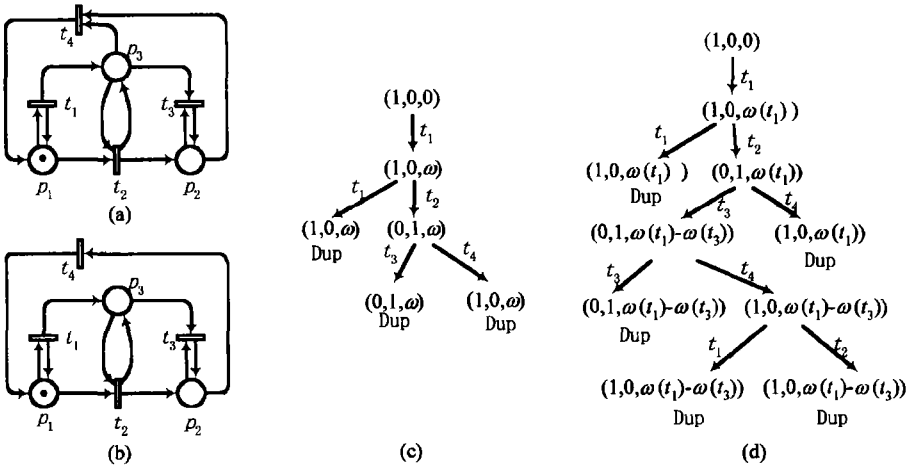

Fig. 1 Place/transition nets and corresponding coverability trees. (a) Petri net N_1 with a deadlock; (b) lived Petri net N_2; (c) ordinary coverability tree of both N_1 and N_2; (d) coverability tree of both N_1 and N_2 constructed in Yuan's way.

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different, they have the same CT (Fig. 1(c)). N_1 has a deadlock e.g. firing sequence t_1, t_2, t_3, while N_2 has not any deadlock. Such a problem results from the fact that for any integer a, $\omega + a = \omega$, i.e. ω cannot tell the increment or decrement of the number of tokens. For example, the number of tokens in p_3 decreases by 1 each time when t_3 is fired. Repeating firing t_3 will eventually result in a marking $M_1 = (0,1,0)$. In N_1, there is no transition enabled at M_1; but in N_2, t_4 is enabled at M_1. However, this cannot be reflected in their CTs.

Figure 1(c) is the CT of both N_1 and N_2 obtained by Yuan's method, which can reveal the change of token number in a CT when using symbol ω[1]. In this method, in order to reflect the decreasing number of token in p_3 after firing t_3, the symbol $\omega_1(t_1) - \omega(t_3)$ is used to replace symbol ω. However, this kind of CT still cannot distinguish the structure of N_1 and N_2 because it cannot extend the nodes containing symbol $\omega(t_1) - \omega(t_3)$. In this note a solution to this problem is presented for a subclass of P/T-nets, namely marked nets (MNs).

1 Exhaustive coverability tree

A marked net is also called an ordinary Petri net[2]. It is such a P/T-net that all its arc weights are either 0 or 1[3]. Many subclasses of P/T-nets such as marked graphs and free-choice nets are all considered as MNs.

The problem is solved by combining some information with ω as follows. Suppose that each node in a CT containing ω (with respect to place s) and the transition that yields this node is t.

(i) Mark ω^{+1}(abbreviated to ω^+) instead of ω if a token is added to s after firing t.

(ii) Mark ω^{-1}(abbreviated to ω^-) instead of ω if a token is removed from s after firing t.

(iii) Mark ω^0 or simply ω if the number of tokens in place s will not change after firing t.

This kind of coverability trees is called exhaustive coverability tree (ECT) in this note, because more details about the change of token number are included and a node containing ω can be extended exhaustively, and ω^{-1}, ω^0 and ω^{+1} have the same properties as ω. So we call them ω-components of a node in an ECT.

Algorithm (Constructing the ECT G for N).

Let N be a marked net with infinite capacities and let M_0 be its initiate marking. Let W_t denote a vector $S \to \mathbb{Z}$ such that $W_t(s) = W(t,s) - W(s,t)$. Write $G_0, G_1, \cdots, G_i, \cdots$, where $G_i = (H_i, P_i)$, and H_i is a set of nodes, P_i is a set of arcs, L_i is a set of arc labels, and let $P_i \subseteq H_i \times L_i \times H_i$ be a sequence of trees constructed according to the following steps.

(1) Let $i = 0$, $H_0 = \{M_0\}$, $P_0 = \emptyset$, such that $G_0 = (\{M_0\}, \emptyset)$. Tag Mo "New". Let $i = 1$, go to Step (2).

(2) While "New" nodes exist in $G_i = (H_i, P_i)$, do the following steps.

(2.1) Select a "New" node $E_i \in H_i$.

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(2.2) If $E_i \in H_{i-1}$, then tag E_i "Dup" (meaning duplicate) and go back to (2.1) to select another "New" node.

(2.3) If there does not exist $t \in T$, and t is enabled at E_i; then tag E_i"Dead", and go back to (2.1) to select another "New" node.

(2.4) While there exists $t \in T$, and t is enabled at E_i, do the following.

(2.4.1) Add a new node $E_{i,0}$. Note that $E_{i,0}$: $S \rightarrow \{N + \omega^{+1} + \omega^{-1} + \omega^0\}$ is a marking function, and $E_{i,0}(s)$ is defined according to the following cases:

(a) if $E_i(s) = \omega^m (m = -1, 0, +1)$, then $E_{i,0}(s) = \omega^{m'}$, $m' = W_t(s)$;

(b) if $E_i(s) \neq \omega^m (m = -1, 0, +1)$ and $\exists E'_i \in H_i$ such that $E'_i \leqslant E_i + W_t$, $E'_i(s) < E_i(s) + W_t(s)$ and there exists a path in G_i from E'_i to E_i, then $E_{i,0}(s) = \omega^{+1}$;

(c) in other cases, $E_{i,0}(s) = E(s) + W_t(s)$.

Tag $E_{i,0}$"New" and let $H_i = H_i \cup \{E_{i,0}\}$ and $P_i = P_i \cup \{(E_i, t, E_{i,0})\}$.

(2.4.2) If $\psi = \{s \mid E_{i,0}(s) = \omega^{-1}\} \neq \emptyset$, let $q = 1$ and do the following.

While there exists a path $p_i = h_0\, t_0\, h_1 \cdots h_r\, t_r\, h_{r+1} \cdots h_{n-1}\, t_{n-1}\, h_n$ from the root to E_i and there also exists $\psi' \subseteq \psi$ satisfying the following conditions,

(a) $\exists\, 1 \leqslant r \leqslant n$ such that $\forall s'_1, s'_2 \in \psi'$: $h_{r-1}(s'_1)$ and $h_{r-1}(s'_2)$ are not ω-components and either (i) for all $k = r, r+1, \cdots, n$, $h_k(s'_1) = h_k(s'_2) = \omega^{m_k} (m_k = -1, 0, +1)$; or (ii) $\forall s_{\Psi'} \in \psi'$: $\exists\, r \leqslant j \leqslant n$: $h_j(s_{\Psi'}) = \omega^+$ and $h_j(s) \neq \omega^+$ for any place s different from $s_{\Psi'}$.

(b) There exists a node h_p in p_i such that $\forall s_1 \in \psi'$, $s_2 \in \psi / \psi'$: $h_p(s_2) = \omega^{m_{s2}}$, and $h_p(s_1)$ is a natural number or $h_p(s_1) = \omega^{m_{s1}}$, m_{s1}, $m_{s2} = -1, 0, +1$ and $m_{s1} < m_{s2}$.

Then add a node $E_{i,q}$ such that $E_{i,q}(s_{\psi'}) = h_{r-1}(s_{\psi'})$ for any $s_{\psi'} \in \psi'$ and $E_{i,q}(s_{/\Psi'}) = E_{i,q-1}(s_{/\Psi'})$ for any $s_{/\Psi'} \notin \psi'$. Tag $E_{i,q}$ "New" and let $H_i = H_i \cup \{E_{i,q}\}$, $P_i = P_i \cup \{(E_i, t, E_{i,q})\}$ and $q = q + 1$.

(2.5) Remove the tag "New" from E_i. Let $H_{i+1} = H_i$, $P_{i+1} = P_i$, $G_{i+1} = (H_{i+1}, P_{i+1})$. Let $i = i + 1$, repeat Step (2) until there is no "New" node in G_i.

(3) $G = G_i$.

Figure 2 shows an example of ECT construction by the algorithm. Fig. 2(c) is the ECT of the net shown in Fig. 2(a). Compared with its CT shown in Fig. 2(b), its ECT is rather complicated, but gives us more information about the change of markings of the net.

2 Detecting deadlock by exhaustive coverability tree

In this paper, deadlock is defined as a marking in which no transition is enabled.

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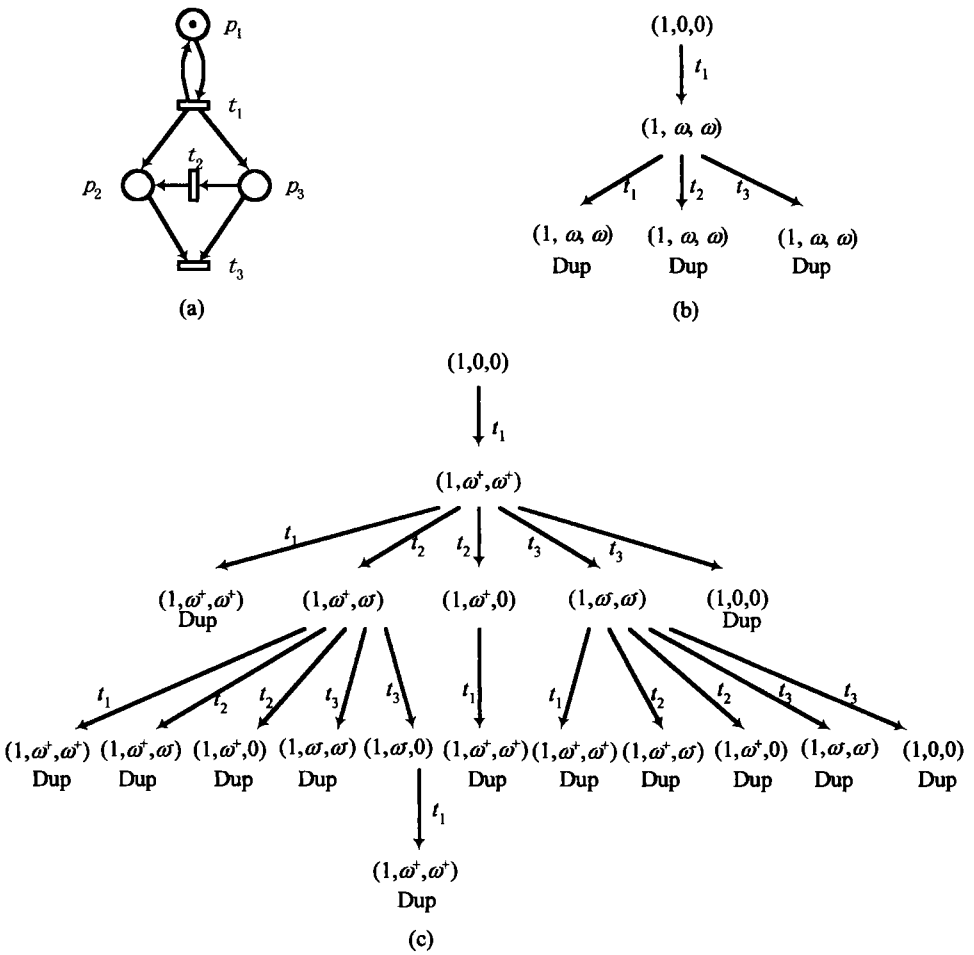

Fig. 2 Marked net N_3 (a) and its coverability tree (b), and exhaustive coverability tree (c).

Definition (Deadlock). A marking $M \in M_0[>$ of a P/T-net is called a deadlock if and only if $\forall t \in T$: t is not enabled.

Theorem[1]**.** Let $G = (H, P)$ be the ECT of some marked net N. G has a deadlock if and only if $\exists E \in H$: E is tagged "Dead".

Go back to the problem presented at the beginning of this paper. Fig. 3(a) is the ECT of N_1 (Fig. 1(a)) and Fig. 3(b) is the ECT of N_2(Fig. 1(b)). There are two nodes (representing the same marking (0,1,0)) tagged "Dead" in Fig. 3(a). This means that N_1 has a deadlock (0,1,0). There is no deadlock in Fig. 3(b) since there is no node tagged "Dead".

3 Conclusion

In this paper, a new kind of coverability tree for marked nets, called exhaustive coverability tree, has been proposed. This kind of coverability tree can reflect the increment and decrement of

1) LU, Y. Q. Petri nets theories and their applications in telecommunications system, Ph. D. Dissertation, South China University of Technology, 1996

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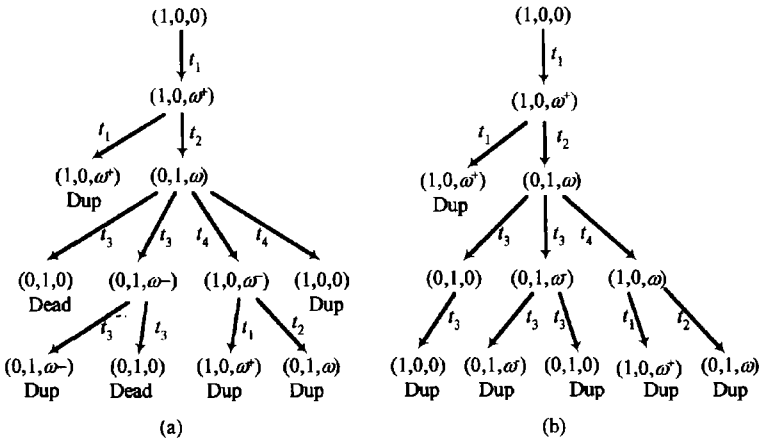

Fig. 3 Exhaustive coverability trees. (a) Exhaustive coverability tree of N_1 shown in Fig. 1(a); (b) exhaustive coverability tree of N_2 shown in Fig. 1(b).

the token number even though the symbol ω is used. It is shown that an ECT can be used to detect deadlock alone. However, an ECT is more complicated than an ordinary coverability tree of the same net. Explosion problems may arise when a marked net is complicated. Simplification techniques can be used to reduce the size of ordinary coverability trees[4,5].

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